

PROBLEM SET 5. DUE TUESDAY, 12 SEPTEMBER

Reading. *Quick Calculus*, pp. 138–142; 148–167.

Supplementary reading. Simmons, Sections 5.1–5.3.

1. (2pts) Approximate the following numbers, using the tangent line approximation.
 - (a) $\sqrt[3]{28}$
 - (b) $\sqrt{102}$
2. (2pts) Find the Taylor series (at $x = 0$) for $f(x) = \frac{1}{1-x}$.
3. (4pts) A sphere of radius r has volume

$$V(r) = \frac{4}{3}\pi r^3,$$

and surface area

$$A(r) = 4\pi r^2.$$

Approximate the volume and surface area of a sphere of radius 7.02cm. You can check your answer by using a calculator to compute the volume and surface area exactly.

4. (4pts) Compute the following integrals.
 - (a) $\int x^3 dx$
 - (b) $\int \sin(x) dx$
 - (c) $\int e^x dx$
 - (d) $\int \sqrt{x} dx$
5. (4pts) Compute the following integrals by substitution, using the substitution given.
 - (a) $\int \sqrt{5+7x} dx$, $u = 5 + 7x$
 - (b) $\int \frac{2x}{\sqrt{3+x^2}} dx$, $u = \sqrt{3+x^2}$
 - (c) $\int 2xe^{x^2} dx$, $u = x^2$
 - (d) $\int \frac{dx}{(x-4)^5}$, $u = x - 4$

6. (4pts) Integrals satisfy

$$\int (f(x) + g(x)) dx = \left(\int f(x) dx \right) + \left(\int g(x) dx \right),$$

just like derivatives do. Again like derivatives, they do **not** satisfy a simple product rule:

$$\int (f(x) \cdot g(x)) dx \neq \left(\int f(x) dx \right) \cdot \left(\int g(x) dx \right),$$

Check that this is indeed not true by using $f(x) = x$ and $g(x) = x$, and computing both sides of the above equation.